Tilt stability of rotating current rings with resistive conductors

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The combined effects of rotation and resistive passive conductors on the stability of a rigid current ring in an external magnetic field are studied. Numerical and approximate analytical solutions to the equations of motion are presented, which show that the ring is always tilt unstable on the resistive decay time scale of the conductors, although rotation and eddy currents may stabilize it over short times. Possible applications of our model include spheromaks which rotate or are encircled by energetic particle rings.

I. INTRODUCTION

A major difficulty for the confinement of plasma in the spheromak configuration[1] is the tendency of the system to tilt. The tilting instability can be understood qualitatively by noting that the magnetic moment of the toroidal current distribution is antiparallel to the vacuum field imposed for confinement. It is well known that this position is a potential energy maximum.

A system of passive coils has been proposed to stabilize the tilt modes.[2] However, these modes are expected to be unstable on the time scale in which currents decay in the coils.[3] Stabilization of the plasma by an energetic particle ring has been suggested.[4] The stabilizing effect of rotation by the plasma itself has also been investigated.[5,6]

The purpose of this paper is to study the combined effects of rotation and resistive passive conductors on the tilt instability. There are different physical situations to which our calculations apply: If rotation is slow compared to other dynamical times, then the stabilizing effect of rotation is weak, and stabilization must be provided by passive coils. We can then ask whether the fluctuating emf applied to the coils by the precessing, tilting system can sustain the eddy currents. On the other hand, if rotation is rapid enough for dynamic stabilization, torques exerted by the resistive conductors may modify the precession. We consider the case of slow rotation to be probably most applicable to a plasma, and the case of fast rotation to be most applicable to an energetic particle ring.

We consider a simple model problem in which a rigid ring of fixed current rotates and nutates in a uniform external field. The ring is surrounded by a set of resistive, passive coils, in which stabilizing currents are induced when the ring tilts. Our simple model for the plasma neglects internal dynamics and shape changes that can accompany a tilt instability,[7,8] but provides a useful framework for understanding basic effects.

In Sec. II coupled equations are derived for the motion of the ring and the induction of the field, and in Sec. III numerical and approximate analytical solutions to the equations of motion are presented. We find that, although precession of the ring does inhibit the decay of currents in the coils, the precession is eventually halted by a torque about the vertical. This torque is inherently caused by finite coil resistivity. Once the precession ceases and the eddy currents decay, the ring rapidly loses equilibrium.

In Sec. IV we summarize our results and briefly discuss their implications for spheromak and other systems.

II. DYNAMICAL EQUATIONS

The torque on a circuit with magnetic moment $\mathbf{m}$ in a uniform magnetic field $\mathbf{B}$ is

$$\frac{d\mathbf{L}}{dt} = \mathbf{m}\times\mathbf{B}. \tag{1}$$

We adopt this as the basic equation of motion. Corrections caused by field nonuniformity are mentioned below.

Euler angles are convenient dynamical variables for the ring. We follow the notation of Goldstein[9]: $\hat{z}'$ is a body-fixed axis perpendicular to the plane of the ring, and the tilt angle θ is the angle between the $\hat{z}'$ and the space-fixed $\hat{z}$ axis. The intersection of the ring plane and the space x-y plane is the line of nodes, and the precession angle ϕ is the angle between the line of nodes and the space x axis. The angle ψ is the angle between the line of nodes and the x' axis rotating with the ring.

Equations of motion equivalent to Eq. (1) can be derived from a Lagrangian in which the potential energy is $U = -\mathbf{m}\cdot\mathbf{B}$. In terms of the Euler angles,

$$\mathbf{m} = m\hat{z}' = m(\hat{x}\sin\theta\sin\phi - \hat{y}\sin\theta\cos\phi + \hat{z}\cos\theta). \tag{2}$$

The field $\mathbf{B}$ consists of an imposed negative vertical field $-\hat{z}B_0$ plus the field caused by induced currents in the surrounding passive coils and can be written

$$\mathbf{B} = \hat{x}B_h\sin\beta - \hat{y}B_h\cos\beta + \hat{z}(B_v - B_0), \tag{3}$$

where $B_h\sin\beta$, $B_h\cos\beta$, and B_v are the $\hat{x}, \hat{y}, \hat{z}$ components of the induced field. The magnitudes B_h and B_v and the angle β are determined self-consistently below in terms of given coil specifications and the dynamics of the tilting ring.

The Lagrangian is then

$$L = \tfrac{1}{2}(\dot\theta^2 + \dot\phi^2\sin^2\theta) + (\dot\psi + \dot\phi\cos\theta)^2 + \omega_0^2(B_v/B_0 - 1)\times\cos\theta + \omega_0^2(B_h/B_0)\sin\theta\cos(\phi-\beta), \tag{4}$$

where we have normalized by dividing by $I/4$, a horizontal component of the inertia tensor, and

$$\omega_0^2 \equiv 4mB_0/I.$$

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Lagrange's equations are

$$\dot{p}_\psi = 2(\dot{\psi} + \dot{\phi}\cos\theta)^{\cdot} = 0; \quad p_\psi = 2\omega = \text{const}; \tag{5a}$$

$$\dot{p}_\phi = (\dot{\phi}\sin^2\theta + 2\omega\cos\theta)^{\cdot} = -\omega_0^2(B_h/B_0)\sin\theta\sin(\phi - \beta); \tag{5b}$$

$$\dot{p}_\theta = \ddot{\theta} = \dot{\phi}^2\sin\theta\cos\theta - 2\omega\dot{\phi}\sin\theta - \omega_0^2(B_v/B_0 - 1)\sin\theta + \omega_0^2(B_h/B_0)\cos\theta\cos(\phi - \beta). \tag{5c}$$

To solve these equations we need an explicit expression for the field produced by the passive coils. A completely realistic calculation would depend explicitly on details of the coil structure and would lead to a spatially nonuniform field. This is more than we need, because our equation of motion assumes a uniform field. In view of this and to preserve some generality in our calculation, we will make a very simple model of the field which should embody most of the necessary physics.

We assume that the passive coils are circular and have negligible mutual inductance so that the current in each coil is driven only by the tilting of the current ring. We calculate the field produced by one such coil, then sum over all coils. We do this in two limiting approximations: a far field case, in which the current loops interact like dipoles, and a near field approximation in which the ring and coils are separated by a distance less than their radii. Let the vector from the center of the current ring to the center of the coil be $\hat{n}R$, the normal to the coil be $\hat{n}_c$, a_c be its radius, and $\mathbf{m}_r$ and $\mathbf{m}_c$ be the ring and coil magnetic moments (see Fig. 1). Then the flux Φ through the coil because of the ring current is given in general by the product of the ring current and the mutual inductance of the two circuits. In our model

$$\Phi = m_r \pi a_c^2 \mathbf{b}\cdot\hat{z},$$

and the field at the plasma caused by the induced coil current is

$$\mathbf{B} = \mathbf{b} m_c,$$

where

$$\mathbf{b} = 3(\hat{n}\cdot\hat{n}_c)(\hat{n} - \hat{n}_c)/R^3 \quad \text{if } a_c \ll R, \tag{6a}$$

and

$$\mathbf{b} = 2\hat{n}_c/a_c^3 \quad \text{if } a_c \gg R. \tag{6b}$$

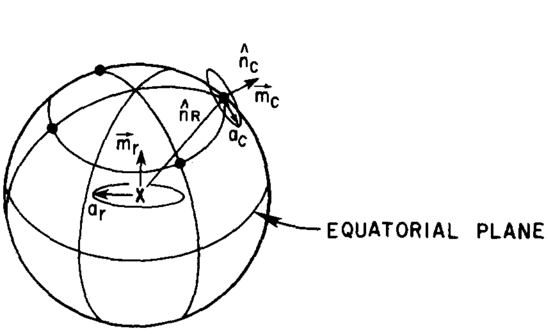

FIG. 1. Location of passive conductors (●) relative to current ring (×). Center positions are shown.

The magnetic moment of the coil is found by solving the circuit equation for the current I_c in the coil:

$$\frac{dI_c}{dt} + \gamma I_c = -\frac{c}{L}\frac{d\Phi}{dt}.$$

Here L is the self-inductance of the coil and $\gamma \equiv R/L$ is the resistive decay rate. The solution for I_c is

$$I_c(t) = \frac{-cm_r\pi a_c^2}{L}\mathbf{b}\cdot\int_0^t dt'\exp[\gamma(t'-t)]\frac{dz'(t')}{dt'},$$

and the field caused by the coil is

$$\mathbf{B} = -(m_r/L)\pi^2 a_c^4 \mathbf{b}(\mathbf{b}\cdot\mathbf{w}),$$

where

$$\mathbf{w} \equiv \int_0^t dt'\exp[\gamma(t'-t)]\frac{d\hat{z}'(t')}{dt'}. \tag{7}$$

We now superimpose the fields of a set of coils. Consider a sphere of radius R centered on the current ring. The ring defines the equatorial plane of the sphere. Four identical coils are placed on a parallel of latitude of the sphere and spaced $\pi/2$ apart (see Fig. 1). The angle $\hat{n}\cdot\hat{n}_c$ is the same for each coil. Such a set is a crude model of the figure 8 coils proposed for a spheromak[2]; another set of four coils is placed at equivalent positions on the opposite hemisphere. The ith component of the total field is then

$$B_i = -m_r\frac{\pi^2 a_c^4}{L}\left(\sum_{j=1}^4 b_{ji}^2\right)w_i;$$

the cross terms cancel because of the symmetry of the coil positions.

To calculate the Σb_{ji}^2 sums for each component, we write $\hat{n}$ and $\hat{n}_c$ for the first coil as

$$\hat{n} = \hat{x}\sin\xi\sin\eta - \hat{y}\sin\xi\cos\eta + \hat{z}\cos\xi,$$

$$\hat{n}_c = \hat{x}\sin\zeta\sin\chi - \hat{y}\sin\zeta\cos\chi + \hat{z}\cos\zeta.$$

Then, in the near field approximation,

$$\left.\begin{aligned} \sum_{i=1}^4 b_{ix}^2 &= \sum_{i=1}^4 b_{iy}^2 = \frac{8\sin^2\zeta}{a_c^6}, \\ \sum_{i=1}^4 b_{iz}^2 &= \frac{16\cos^2\zeta}{a_c^6}. \end{aligned}\right\} \quad a_c \gg R. \tag{8}$$

In the far field approximation,

$$\left.\begin{aligned} \sum_{i=1}^4 b_{ix}^2 &= \sum_{i=1}^4 b_{iy}^2 = 2\{3(\hat{n}\cdot\hat{n}_c)^2 + 1 \\ &\quad - [3(\hat{n}\cdot\hat{n}_c)\cos\xi - \cos\zeta]^2\}, \\ \sum_{i=1}^4 b_{iz}^2 &= 4[3(\hat{n}\cdot\hat{n}_c)\cos\xi - \cos\zeta]^2(R^6)^{-1}. \end{aligned}\right\} \quad a_c \ll R. \tag{9}$$

Notice that if $\hat{n} = \hat{n}_c$, expressions (8) and (9) coincide when $a_c = R$.

To make contact with the notation for $\mathbf{B}$ introduced in Eq. (3) we let

$$\alpha^2 B_0 \equiv \frac{m_r\pi^2 a_c^4}{L}(\Sigma b_x^2) = \frac{m_r\pi^2 a_c^4}{L}(\Sigma b_y^2), \tag{10}$$

$$\beta^2 B_0 \equiv \frac{m_R\pi^2 a_c^4}{L}(\Sigma b_z^2)$$

define the induced horizontal and vertical field strengths.

We then integrate **w** by parts and use Eq. (2) for $\hat{z}'$ to derive explicit expressions for B_v, $B_h \cos\beta$, and $B_h \sin\beta$. This results in the final equations of motion [which are Eqs. (5b) and (5c) rewritten using the calculated induced field],

$$\ddot{\theta} = \dot{\phi}^2 \sin\theta\cos\theta - 2\omega\dot{\phi}\sin\theta + \omega_0^2 \sin\theta + \omega_0^2(\beta^2 - \alpha^2)\sin\theta\cos\theta + \gamma\omega_0^2 \int_0^t dt' \exp[\gamma(t'-t)] \times \{\alpha^2 \cos\theta(t)\sin\theta(t')\cos[\phi(t)-\phi(t')] - \beta^2 \sin\theta(t')\cos\theta(t')]\}, \quad (11a)$$

$$\dot{p}_\phi = -\alpha^2\omega_0^2\gamma \int_0^t dt' \exp[\gamma(t'-t)]\sin\theta(t')\sin[\phi(t) - \phi(t')]. \quad (11b)$$

The parameters α^2 and β^2 in Eqs. (11a) and (11b) completely describe the effect of passive coils in our model. These constants are to be calculated from Eq. (10) and Eqs. (8) or (9), depending on whether the coils are close to or far from the current ring.

III. TILT DYNAMICS

Equation (11) can be solved for the tilt angle θ and tilt orientation ϕ as functions of time. In the absence of dissipation, p_ϕ is conserved and the dynamics is analogous to a sleeping top in gravity.[5] Referring back to Eq. (5), and noting that when $\gamma = 0$ the vector **w** defined in Eq. (7) is simply $\hat{z}'$ itself, we see that when $\gamma = 0$ the horizontal components of the induced field are exactly aligned with the tilt direction. The presence of resistivity causes the induced field to be slightly out of phase with the tilt direction, leading to a torque on p_ϕ.

There are two widely separated time scales in the problem: the dynamical time scale, of order ω_0^{-1}, and the resistive time scale, of order γ^{-1}; typically $\gamma/\omega_0 \approx 10^{-2}$–$10^{-3}$. We will first study the motion in the ideal limit, with $\gamma = 0$, and then see how the configuration evolves when resistivity is present.

When $\gamma = 0$, there is an energy integral for the system

$$E = \tfrac{1}{2}\dot{\theta}^2 + V(\theta),$$

where

$$V(\theta) = \tfrac{1}{2}[(p_\phi - 2\omega\cos\theta)^2/\sin^2\theta] + \omega^2 + \omega_0^2\cos\theta + (\omega_0^2/4)(\beta^2 - \alpha^2)\cos 2\theta, \quad (12)$$

and $\dot{\phi}$ has been eliminated in favor of θ and p_ϕ. The motion in θ is periodic, and the system is usefully "stable" if the outer turning point in θ is small.

We first consider the potential V when p_ϕ and ω are zero. Then, V has extrema at $\theta = 0$, $\theta = \pi$, and $\theta = \cos^{-1}[1/(\alpha^2 - \beta^2)]$. If $\beta^2 - \alpha^2 < -1$, the points $\theta = 0$ and $\theta = \pi$ are minima, while if $\beta^2 - \alpha^2 > 1$, they are maxima, and the minimum of the potential well occurs for $\theta > \pi/2$. If $|\beta^2 - \alpha^2| < 1$, $\theta = 0$ is a maximum and $\theta = \pi$ a minimum.

These results show that the best situation for stability is one in which $\beta^2 - \alpha^2 < -1$, so that oscillations about zero exist for $E < \omega_0^2/2(\alpha^2 - \beta^2) + \omega_0^2(\alpha^2 - \beta^2)/4$. The induced vertical field, represented by β, is therefore destabilizing, while the induced horizontal field, represented by α^2, is stabilizing. From Eqs. (8), (9), and (10) we see that $\beta^2 \ll \alpha^2$ if the coil position angles ζ and ξ are chosen to be near $\pi/2$. We will henceforth assume that this is the case and drop β^2. The β^2 terms do not interact with the rotation, which is our primary interest here.

The result that $\alpha^2 > 1$ for stability is related to a similar, linearized analysis[10] in which the effect of a nonzero field index $n_i \equiv -(r/B)\partial B/\partial r$ was considered. In order for us to take the field index into account, we must calculate the torque on the current ring because of a nonuniform field; this results in the third term in $V(\theta)$ being replaced by $2\omega_0^2(1 - n_i)/(n_i + 2)\cos\theta$ and is inessential for our calculation.

We now consider the effect of rotation. Even a small value of ω or p_ϕ changes the problem qualitatively because centrifugal barriers appear at $\theta = 0$ and $\theta = \pi$. We can make this case analytically tractable by considering E and V for small angles. Then

$$E' - \omega p' = \tfrac{1}{2}\dot{\theta}^2 + p'^2/2\theta^2 + [\omega^2 + (\alpha^2 - 1)\omega_0^2](\theta^2/2), \quad (13)$$

$$p' \equiv p_\phi - 2\omega \approx \theta_0^2(\dot{\phi}_0 - \omega),$$

$$E' \equiv E - \omega^2 - \omega_0^2 + \alpha^2\omega_0^2/4,$$

where θ_0 and $\dot{\phi}_0$ are the initial values of θ and $\dot{\phi}$. In making this expansion, we have used the fact that p' is of order θ_0^2.

It is clear from the form of Eq. (13) that a potential minimum near $\theta = 0$ exists only if $[\omega^2 + (\alpha^2 - 1)\omega_0^2] > 0$. This is consistent with other results on the stabilization of rings by rotation[4,5,11] when conductors are absent; $\omega^2 > \omega_0^2$ for stability.

We can exactly solve the equation of motion derived from Eq. (13). The result is

$$\theta^2 = \frac{(E' - \omega p')}{\omega'^2} + \left(\frac{(E' - \omega p')^2}{\omega'^4} - \frac{p'^2}{\omega'^2}\right)^{1/2} \cos 2\omega' t, \quad (14)$$

$$\omega'^2 \equiv \omega^2 + (\alpha^2 - 1)\omega_0^2$$

(we have chosen $\dot{\theta}_0 = 0$). Notice that in the limit $\omega \to 0$, the frequency of the motion is twice what it would be in the nonrotating case; this is caused by reflection by the barrier at $\theta = 0$.

The precession rate of the system is

$$\dot{\phi} = (p' + 2\omega - 2\omega\cos\theta)/\sin^2\theta \approx \omega + p'/\theta^2. \quad (15)$$

We now consider the effect of dissipation. Without rotation, the problem is similar to that studied for the shift mode in tokamaks.[3] When $\alpha^2 > 1$ (the dynamically stable regime) there are two oscillatory modes which are damped on the L/R time scale γ^{-1}, and one mode which grows on the L/R time. This can be seen from our equations by a small angle approximation in Eq. (11a). The equation can then be integrated exactly and has three exponential solutions; $\theta \sim \exp \nu t$, where, to first order in γ/ω_0,

$$\nu_{1,2} = \pm i\omega_0(\alpha^2 - 1)^{1/2} - \alpha^2\gamma/2(\alpha^2 - 1), \quad (16)$$

$$\nu_3 = \gamma/(\alpha^2 - 1).$$

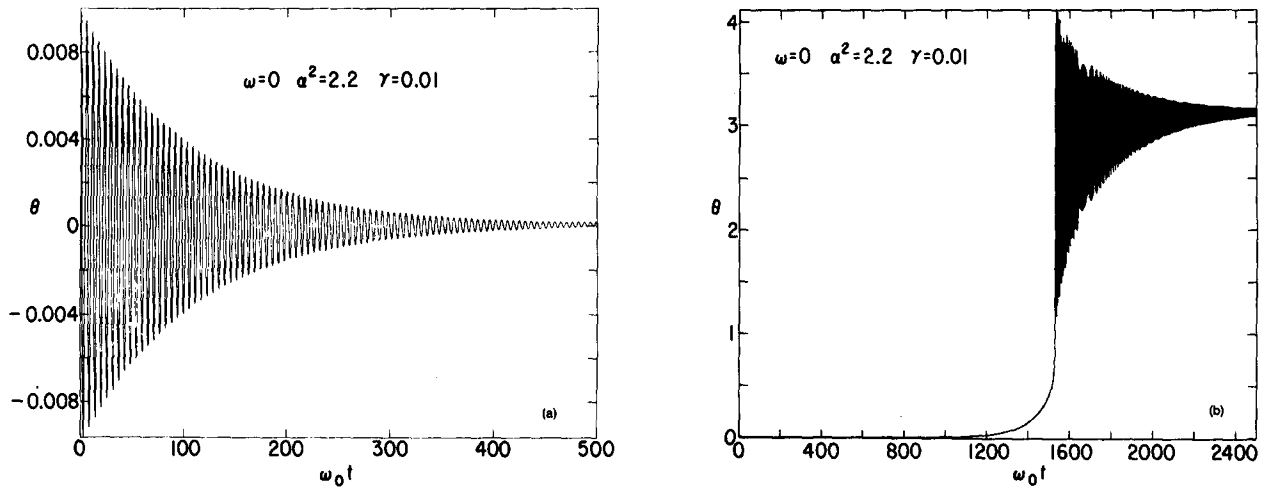

FIG. 2. Evolution of the tilt angle θ for a resistive, nonrotating case. (a) Short time evolution of tilt angle θ. Rotation absent ($\omega = 0$). (b) Long time evolution of θ showing loss of stability.

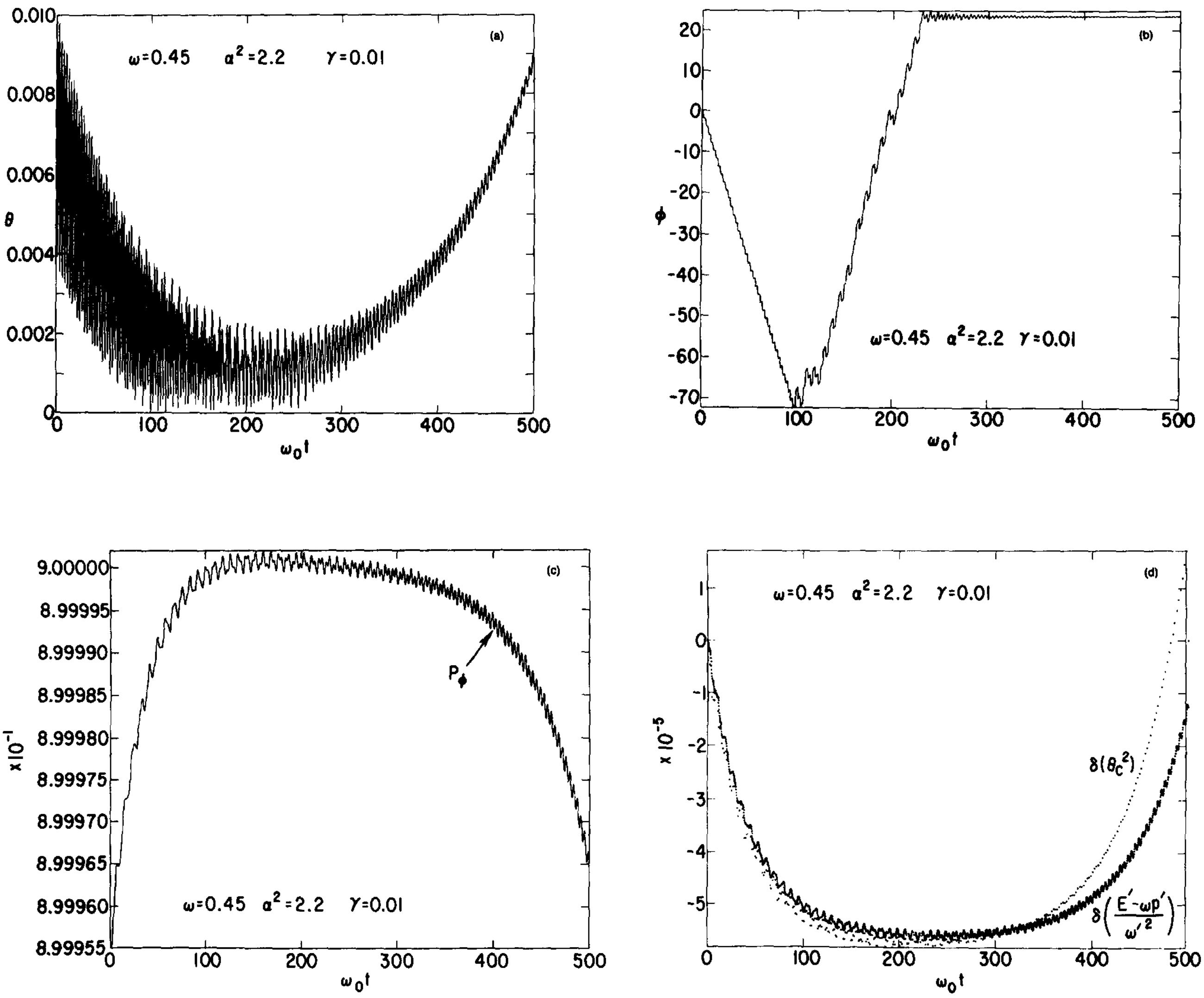

FIG. 3. Evolution of the tilt angle θ when both rotation and resistivity are present. (a) Short time evolution of θ when rotation and resistivity are present, showing high-frequency oscillation about centroid drift. (b) Evolution of precession angle ϕ. Eventually ϕ becomes nearly constant and the dissipative terms become effective. (c) The evolution of p_ϕ. (d) Agreement between full dynamics and small angle approximation Eq. (14). Here $\delta(x)$ refers to the change in x from its initial value.

The result of a numerical integration of Eq. (11) with $\omega = 0$ is shown in Fig. 2. In Fig. 2(a), short time evolution is shown; θ oscillates about zero with period $2\pi[\omega_0^2(1-\alpha^2)]^{1/2}$. In Fig. 2(b) the long time evolution is shown; θ rapidly climbs to π (i.e., goes unstable) and executes damped oscillations about its new equilibrium. In the case depicted in Fig. 2, $\dot{\theta}$ is initially zero. It can be shown that this choice almost completely eliminates the unstable mode from the perturbation, which explains why the growth time is substantially longer than γ^{-1}.

We now consider the full problem, with dissipation and rotation. Figures 3(a)–3(d) show the evolution of a typical system over the first five resistive times. The initial conditions chosen were $\theta_0 = 10^{-2}$, $\dot{\theta}_0 = 0$, $\dot{\phi} = 0$, $\phi_0 = 0$. The small angle approximation is clearly adequate for the short times shown here.

Figure 3(a) shows that the motion in θ consists of high-frequency oscillations at frequency $2\omega'$ superimposed on a centroid θ_c that first drifts in toward zero and then begins to climb. This climb is the onset of a rapid increase to the vicinity of π, as shown in Fig. 4. We can understand this by considering Figs. 3(b)–3(d).

Initially, $\langle \dot{\phi} \rangle$, the mean value of $\dot{\phi}$, is negative, as predicted by Eq. (15) for our initial conditions. This means that $p_{\dot{\phi}}$ increases [see Eq. (11b)], and θ_c decreases as shown by Eq. (14). When p_ϕ reaches a value near 2ω, $\langle \dot{\phi} \rangle$ becomes positive and $\dot{p}_\phi$ becomes negative. This feedback between p_ϕ and $\dot{\phi}$ leads to an oscillation of p_ϕ about 2ω and of $\langle \dot{\phi} \rangle$ about zero. Eventually this oscillation is damped and ϕ becomes nearly constant. Once ϕ is constant the integrand in the dissipative term of Eq. (11a) is no longer oscillatory, and the dissipative term increases rapidly. Then, once the eddy currents decay, θ increases rapidly. Figure 3(d) shows that the motion of the centroid θ_c^2 is quite well represented by Eq. (14). Thus rotation does not prevent the decay of the eddy currents. Instead, the small phase shift between the tilt and field directions eventually brakes the precession so that the eddy currents decay. The system evolves rapidly to a damped oscillation near $\theta = \pi$ (Fig. 4).

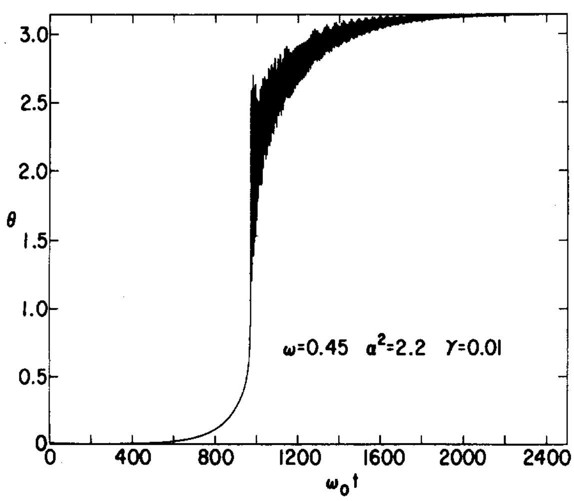

FIG. 4. Long time evolution of θ. Eventually the system point falls over the potential hill and executes damped oscillation near $\theta = \pi$.

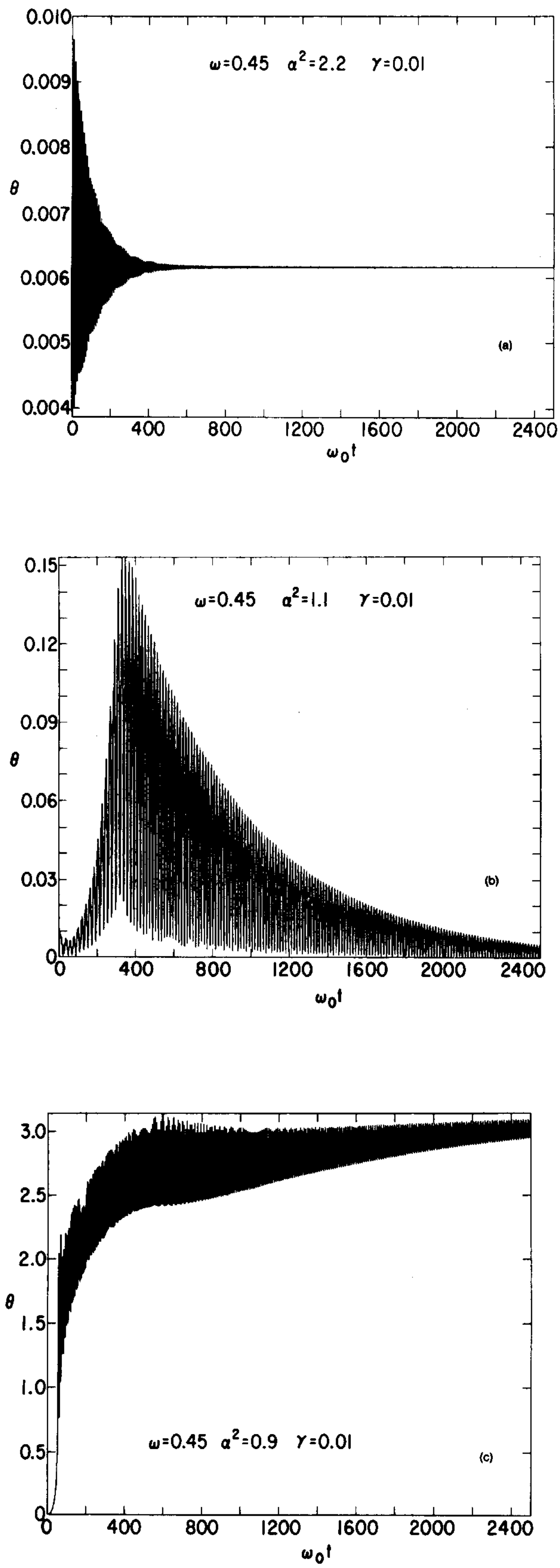

FIG. 5. Numerical experiments to elucidate the role of resistive torques and drag forces. (a) Effect of artificially keeping p_ϕ constant. (b) Effect of allowing p_ϕ to evolve, but removing the dissipative term from the acceleration in θ. Here $\alpha^2 = 1.1$ and is large enough to stabilize the tilt without precession. (c) The same as (b), but $\alpha^2 = 0.9$, too small to stabilize tilt without precession.

We have tested our picture of the instability by modifying the equations of motion. Figure 5(a) shows the effect of removing the dissipative term in Eq. (11b), so that p_ϕ is artificially kept constant. Evidently θ remains bounded, executing damped oscillations around θ_c. Figures 5(b) and 5(c) show the effect of allowing resistivity to act on p_ϕ, but removing the resistive term in Eq. (11a). In Fig. 5(b), $\alpha^2 > 1$, so the horizontal field is large enough to stabilize the system without rotation, and θ_c remains bounded. In Fig. 5(c) $\alpha^2 < 1$ but $\omega'^2 > 1$. In this case θ_c grows large once the precession is braked by resistive torques, because the horizontal field is not strong enough to provide stability.

In Fig. 6 we show the results of a parameter study in which the growth time for the instability, defined as the time at which $\theta = \pi/4$, is computed as a function of α^2 and ω. The contours show that the growth times are much more sensitive to α^2 than to ω.

We close this section with a numerical exercise in which we calculate typical values of ω_0^2 and α^2 for the S-1 Spheromak.[12]

The dynamical frequency $\omega_0 = (4mB_0/I)^{1/2}$ *is* $(2mB_0/MR^2)^{1/2}$ for a thin ring of mass M and radius R. We relate B_0 and m by an approximation to the Shafranov formula[13]; $B_0 \sim m/\pi R^3 \ln 8R/a$. Taking the current, radius, and number density in the ring to be 200 kA, 50 cm, and 10^{14} cm^{-3}, respectively, we find $\omega_0 \approx 1.10^6$ sec^{-1}.

To calculate α^2 we take the radius of each coil to be 40 cm, assume its normal direction ζ is $\pi/4$, and that the coil and plasma centers are 40 cm apart. We compute the self-inductance of the coil assuming its thickness is 2 cm. With the other parameters as above, $\alpha^2 = 4.6$ in the near field approximation and 3.0 in the far field approximation. Equation (16) therefore predicts that the spheromak becomes tilt unstable on a time scale of about two to four times the L/R time of the passive coils. We have assumed two sets of coils, one above and one below the plasma. Our model is too simple to describe the effects of the coils very accurately, but it does show that α^2 can easily be of order unity.

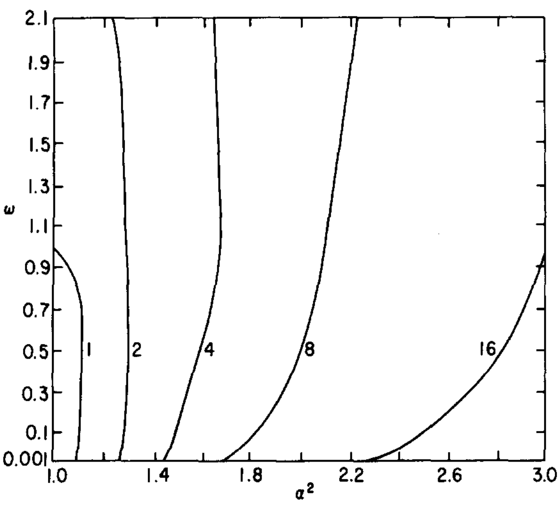

FIG. 6. Contours of constant growth time of the instability in units of γ^{-1}.

IV. DISCUSSIONS AND CONCLUSIONS

We have studied the effects of rotation and resistive, passive conductors on the tilt instability of a rotating current ring in an external magnetic field. Several motivations exist for this problem.

It is known from studies of similar problems[3] that, when passive coils provide dynamic stabilization, the system is unstable on the resistive decay time of the currents in the conductors. It might be expected that decay of the eddy currents could be prevented by the fluctuating emf produced by a tilting, precessing ring.

On the other hand the torques exerted by the resistive coils also affect the precession. This phenomenon is important in any system which rapidly rotates because nearby conductors are never entirely absent.[14] Resistive torques could also be important in systems with rotating nonaxisymmetric modes.

We made a simple model of a rigid ring of fixed current. Deformation of the ring[15] and changes of current[5] are certainly important, but they require a detailed plasma model to compute accurately. It is unlikely that inclusion of either effect would suppress the phase lag between the direction of tilt and the direction of the induced field, because this lag is caused by the finite resistivity of the passive coils. The phase lag is ultimately responsible for the braking of the precession because of the torque it produces on the ring. Only when the ring precesses do the stabilizing eddy currents persist without decay. Thus we conjecture that more complex models would also eventually be destabilized by resistive torques.

The principal conclusions of our analysis are as follows:

(1) Rotation does not prevent decay of the eddy currents. If the coils are perfectly conducting, the induced horizontal field is exactly aligned with the direction of tilt. Resistivity causes a small phase lag in the direction of the induced field. The resulting torque halts the precession and the current in the coil decay typically within less than 20 L/R times.

(2) The horizontal components of the field produced by the coils are stabilizing, while the induced vertical field is destabilizing (on the dynamical time scale). Stabilization is optimized when the coils are placed close to the equatorial plane of the ring, with their normals nearly perpendicular to the ring normal.

(3) If the effect of resistivity on the precession rate (or p_ϕ) is artifically suppressed, the system becomes stable because the eddy currents are prevented from decaying. Our *ad hoc* procedure for maintaining the precession [e.g., see Fig. 5(a)] can, in principle, be replaced by an active feedback system.

Although our model is simple, we believe it provides a framework for understanding the problem. Our results suggest that rotation of the plasma, or a circumferential particle beam, will not stabilize the system to modes on a resistive timescale.

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[1]M. N. Rosenbluth and M. Bussac, Nucl. Fusion **19**, 489 (1979).
[2]S. C. Jardin and U. Christensen, Nucl. Fusion **21**, 1665 (1981).
[3]S. C. Jardin and D. A. Larrabee, Nucl. Fusion **22**, 1095 (1982).
[4]R. N. Sudan and P. K. Kaw, Phys. Rev. Lett. **47**, 575 (1981).
[5]A. Mohri, Jpn. J. Appl. Phys. **19**, L686 (1980).
[6]S. Itoh and K. Itoh, Jpn. J. Appl. Phys. **20**, L287 (1981).
[7]T. Sato and T. Hayashi, Phys. Rev. Lett. **50**, 38 (1983).
[8]T. Hayashi and T. Sato, Phys. Fluids **27**, 778 (1984).
[9]H. Goldstein, *Classical Mechanics* (Addison-Wesley, Reading, MA, 1950).
[10]S. C. Jardin, M. S. Chance, R. L. Dewar, R. C. Grimm, and D. A. Monticello, Nucl. Fusion **21**, 1203 (1981).
[11]J. M. Finn and R. N. Sudan, Nucl. Fusion **22**, 1443 (1982).
[12]M. Yamada, R. Ellis, Jr., H. P. Furth, G. Hart, A. Janos, S. Jardin, F. Levinton, D. Meyerhofer, M. Mimura, C. H. Nam, S. Paul, A. Sperduti, S. Von Goeler, F. Wysocki, P. Young, *Proceedings of 10th International Conference on Plasma Physics and Controlled Nuclear Fusion Research, 1984* (IAEA, Vienna, 1985).
[13]V. D. Shafranov, Rev. Plasma Phys. **2**, 103 (1966).
[14]K. Kupfer, H. H. Fleischmann, and A. Mohri, Bull. Am. Phys. Soc. **29**, 1358 (1984).
[15]C. B. Ruchti and R. V. Lovelace, Phys. Fluids **27**, 1789 (1984).